STATE DA VINCI DECATHLON 2018

CELEBRATING THE ACADEMIC GIFTS OF STUDENTS IN YEARS 5 & 6

MATHEMATICS

TEAM NUMBER _____

1	2	3	4	5	6	7	8	9	10	TOTAL	RANK
/16	/10	/4	/3	/4	/7	/6	/5	/6	/5	/66	

QUESTION 1: CICADA CYCLES (16 MARKS)

It has been found that cicadas hatch with a life cycle that lasts a length always equal to prime numbers, creating unexpected hatching years at the end of each cycle. Some species have 13 years while cicadas in Eastern United States have 17 years. Being prime, the number can't be divided evenly by a smaller number than itself except for 1.

(a) Most other animals, including the predators of Cicadas have life cycles of only 2-8 years. A predator has a life cycle of 4 years, another inspect prey has a life cycle of 6 years and a cicada prey has a life cycle of 7 years. **At the end of 2000** all three creatures completed their life cycle. Create a table and list the years that the next 6 cycles will end for each creature (6 marks)

(b) In which years will there be a predator and prey concluding their cycle together (i.e. hatching)? (3 marks)

(c) Using part (e) explain why cicadas have **evolved to have prime number** life cycles. (2 marks)

(d) A cicada has a life cycle of 13 years. A life cycle ends at the end of 1995. Will the cicada's life cycle end (i.e. they hatch) at the end of 2173? Show your working to explain. (2 marks)

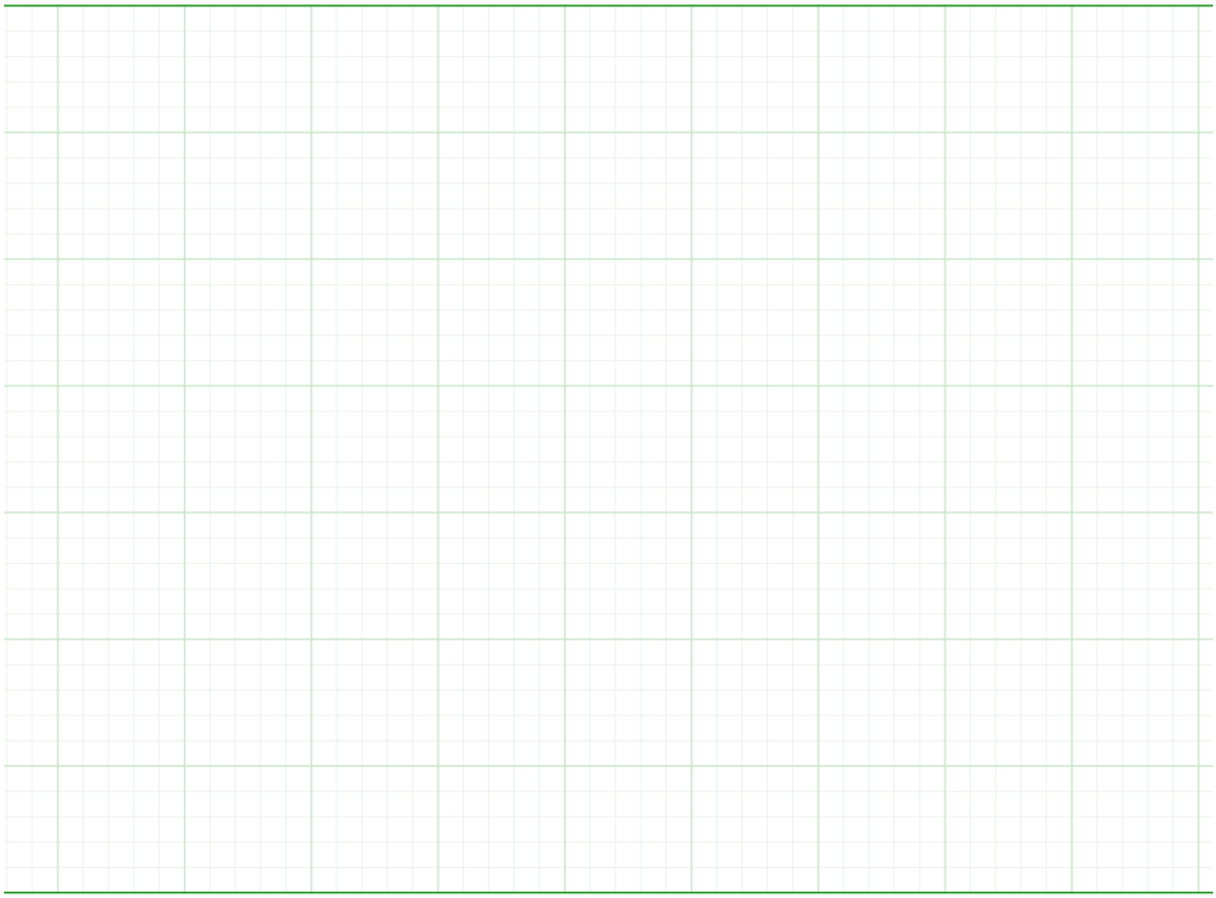
(e) A new insect has been found to have a life cycle of 421 years. Do you think this insect is a cicada? Explain. (3 marks)

QUESTION 2: UNSURPRISING SPONTANEITY (10 MARKS)

Below are a set of results from a test that recorded how many milliseconds it took for individuals of different ages to press a buzzer once a green screen turned red at a random unpredictable time.

Age	Time to respond (milliseconds)
4	84
5	104
7	144
14	284
20	404
30	604
50	1004
55	1104
65	1304
70	1404
72	1444
75	1504

(a) Construct a line graph of this data. Consider including an appropriate title, axis scales and position of values on axis (age vs. time to respond). (4 marks)



(b) By sketching a line that fits through the data, predict how long it would take for a 21-year-old to respond. (2 marks)

(c) By extending the line in (b) predict how long it would take for a 90-year-old to respond. (2 marks)

(d) Is it likely your answer in (c) would accurately reflect reality if a 90-year-old were to participate in the experiment? (2 marks)

QUESTION 3: APPLE CORE (4 MARKS)

10 green and red apples are organised as below:

By switching only two adjacent apples at a time, what is the least number of switches you need to make so that the new arrangement below can be achieved? Explain.

QUESTION 4: COOKIE THIEF (3 MARKS)

It is known that 2 cookies + 2 cookie jars = 1 cookie jar + 12 cookies. How many cookies are in a cookie jar?

QUESTION 5: CLASSROOM CLOWNS (4 MARKS)

Three different teachers have students organised into groups for a group exercise. Mr James has four groups containing 2, 3, 4 and 5 students. Mr Smith has groups of 4, 5, 6 and 7 while Mrs Philips has groups of 6, 7, 8 and 9.

Each teacher must have the same number of students to supervise. Which **one** group should be moved to another teacher? Explain.

QUESTION 6: FRUITY PROBLEM (7 MARKS)

(a) Alex orders apple juice boxes into square containers. If a container is divided with two dividers (each extend from one edge to the other of the square) he can fit 4 juice boxes. If he uses 4 dividers, he can fit 9 juice boxes **without each box** touching each other. Draw a sketch to illustrate each situation (2 marks)

(b) How many dividers are needed to hold 100 milk cartons? (3 marks)

(c) If Alex wants to fit b^2 juice boxes into a square create (without touching) how many dividers are needed? (2 marks)

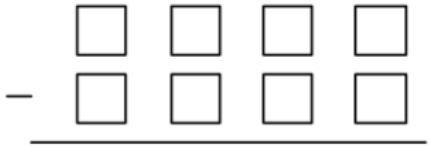
QUESTION 7: PLANNING TO WIN? (6 MARKS)

Hannah has a square crate that can hold 9 cartons of milk. She plays this game with Tane. First she puts a carton in, and then he does. They keep alternating in this way. However, the rule is that no three cartons can be in a line. The winner is the one who puts the last carton in the crate.

If Hannah always goes first, who should always win, Hannah or Tane?

QUESTION 8: MINIMISING DAMAGE (5 MARKS)

Use the digits 0, 1, 2, 3, 4, 5, 6, 7 to find the smallest answer possible in the problem below:



QUESTION 9: SHAPE SHIFTING (6 MARKS)

Peter keeps a piece of string from a parcel that came for his birthday. It is 30 cm long.

He plays with it and makes different shapes all with the same perimeter of 30 cm.

Then he wonders about polygons.

Which has the biggest area: an equilateral triangle, a square or a regular hexagon?

His sister Kiri says that she thinks he can get more area inside a circle with that perimeter.

Which of these figures has the biggest area?

Note: the area of a square = (side length)^2
area of a circle = π x (radius)^2 where the radius is the distance from the centre of the circle to an edge using a straight line.
the area of an equilateral triangle = $\frac{\sqrt{3}}{4} \times (side\ length)^2$

QUESTION 10: AGE OLD DILEMA (5 MARKS)

A Greek mathematician Diophantus developed the following problem:

> *When first the marriage knot was tied between my wife and me,*
> *Her age did mine as far exceed as three plus three does three;*
> *But when three years and half three years we man and wife had been*
> *Our ages were in ratio then as twelve is to thirteen.*

How old were they on their wedding day?