KNOX GRAMMAR SCHOOL

STATE

DA VINCI DECATHLON 2018

CELEBRATING THE ACADEMIC GIFTS OF STUDENTS IN YEARS 7 & 8

da Vinci Decathlon® EXCITE ENRICH CHALLENGE An initiative of Knox Grammar School

MATHEMATICS

TEAM NUMBER _____

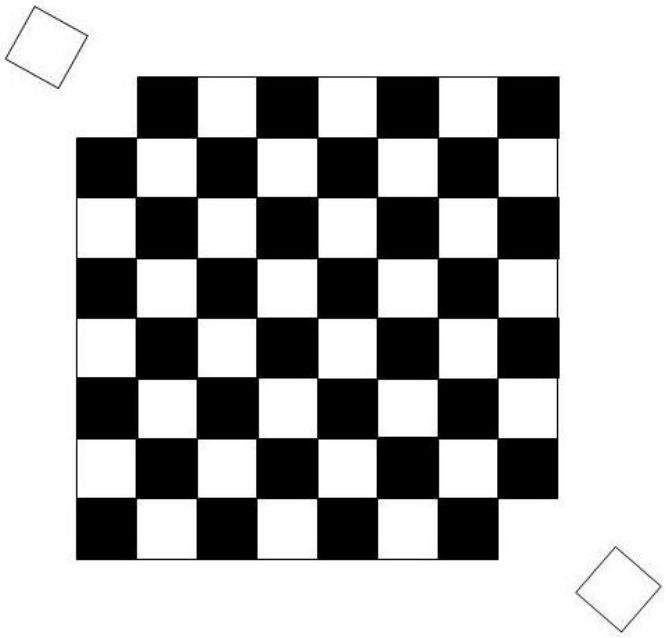
1	2	3	4	5	6	7	8	9	Total	Rank
/4	/6	/6	/4	/8	/4	/6	/4	/8	/50	

QUESTION ONE

THE MUTILATED CHESSBOARD

THE PROBLEM 4 MARKS

Mark is an avid board-gamer. His two favourite games are chess and dominoes. One day, he decides that he would like to combine the two and so he retrieves his chessboard and his 32 dominoes. By stroke of luck, each domino perfectly covers two squares of the chessboard. Therefore, his full collection covers the entire board. However, then he has the idea of **cutting off two corners**, as shown in the diagram below.



Mark **expects** that he will now be able to cover the board with 31 of his 32 dominoes. **Is this possible, and how?** Or, if it is not, **prove that it is impossible.** The dominoes must still cover two full squares each (i.e. **not diagonally**).

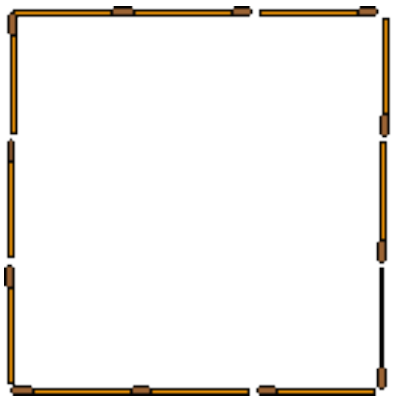
ANSWER SPACE

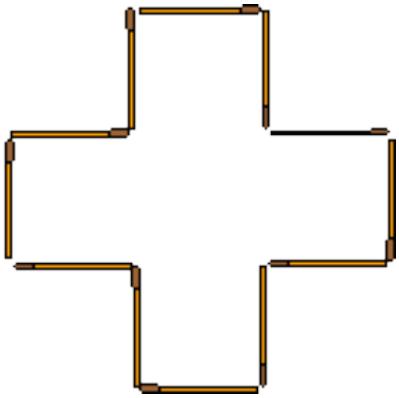
QUESTION TWO

MATCH-ING GAME

THE PROBLEM 6 MARKS

John has twelve matches. Each is one unit in length. He creates polygons such as those below with ease, with areas of nine and five square units respectively. However, he doesn't **expect** to be able to create a polygon of only **four square units,** while using the entire lengths of all twelve matches as with these first two examples.





He does, though, persist, and is hit by a sudden rush of creative inspiration! He is able to create eight elegant polygons, all of four square units.

Your task is to draw three of them below, together with proof that the area of each of them is indeed four units square.

ANSWER SPACE

QUESTION TWO ANSWER SPACE CONTINUED

QUESTION THREE

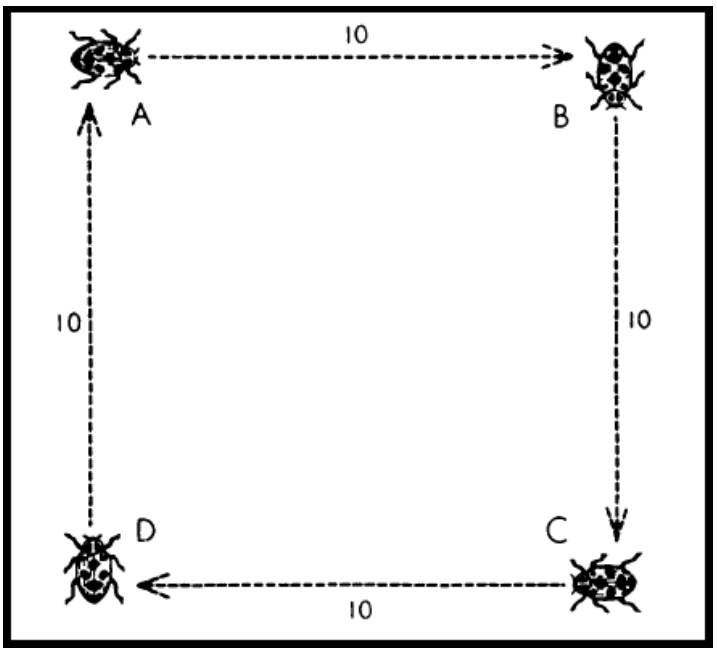
A BUG'S LIFE

THE PROBLEM 6 MARKS

Four bugs occupy the corners of a square which is 10cm long on each side, as shown in the diagram below. The bugs are labelled A, B, C and D. Each bug is chasing the other, meaning that A is chasing B, B is chasing C, C is chasing D and D is chasing A. Every bug is crawling at the same speed.

Although these bugs are highly intelligent and can sense that they are travelling at the same speed as their target, they each do not know that their target is also chasing another bug. Therefore, **they do not expect to ever catch the bug in front of them,** because they anticipate only moving in a straight line.

Will each bug reach their target? If so, will they do so at the same time? **Finally, if so again, how far will each bug crawl until it reaches the one in front of it and why?**



ANSWER SPACE

QUESTION FOUR

DIVIDE, DIVIDE, REPEAT

THE PROBLEM

4 MARKS

A mathematician tells his class to write down a **three-digit numbe**r (i.e. a number between 100 and 999) on a piece of paper. Just some of the numbers that the students choose include 394, 561, 777, 800 and 999. Then, the mathematician tells the class to duplicate their numbers (394394, 561561, etc.) before handing their paper to the person to their left.

The students are to **divide** the number they receive by 7, before passing it to the left again. Then, they **divide** the new number that they have on their desk by 11, and again pass the sheets of paper to their left. Finally, they are told to **divide** this fourth different number in front of them by 13. This time, however, each student is instructed to pass the sheet to their right **three times.**

110	220	341	473	671
121	231	352	484	682
132	242	363	495	693
143	253	374	550	770
154	264	385	561	781
165	275	396	572	792
176	286	440	583	880
187	297	451	594	891
198	330	462	660	990

To every student's amazement, the sheets in front of them now have their **original three-digit number on it. Explain how this is possible** for every three-digit number.

ANSWER SPACE

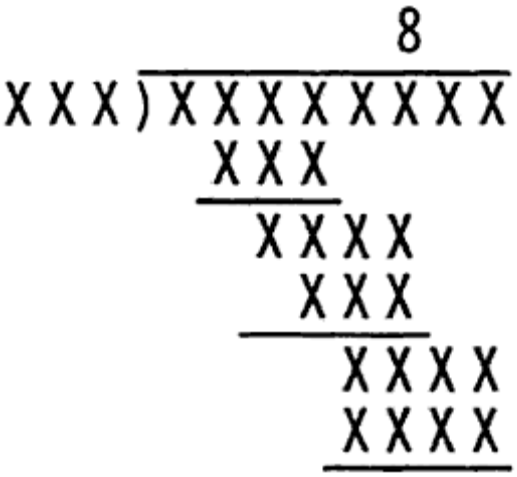
QUESTION FIVE

THE LONESOME 8

THE PROBLEM 8 MARKS

The following is a famous **long-division** problem, attributed to P. L. Chessin of the Westinghouse Electric Corporation. It was published in the April 1954 edition of *The American Mathematical Monthly* and remains the most popular problem ever published in that journal.

There are two ways to solve it. The first is trial and error, which would require testing the entirety of the **81×10^9** possible **permutations** of digits. The second is to realise that, through a few simple mathematical observations, the number of possible answers can quickly be reduced to $(81 \times 10^9)^0$. So, without further ado, **solve** the puzzle!



ANSWER SPACE

QUESTION FIVE ANSWER SPACE CONTINUED

QUESTION SIX

BILL SHOCK

THE PROBLEM

4 MARKS

Emily goes to the bank one day to open an account and deposit a cheque. However, the absent-minded bank teller, who is busy thinking about what to watch on Netflix this evening, switches the dollars and cents values when depositing Emily's money.

Later, Emily buys a 5-cent chocolate from the corner store, but is **shocked** when she checks her bank balance online! She now has **twice** as much as her original cheque.

What was the value of the cheque?

ANSWER SPACE

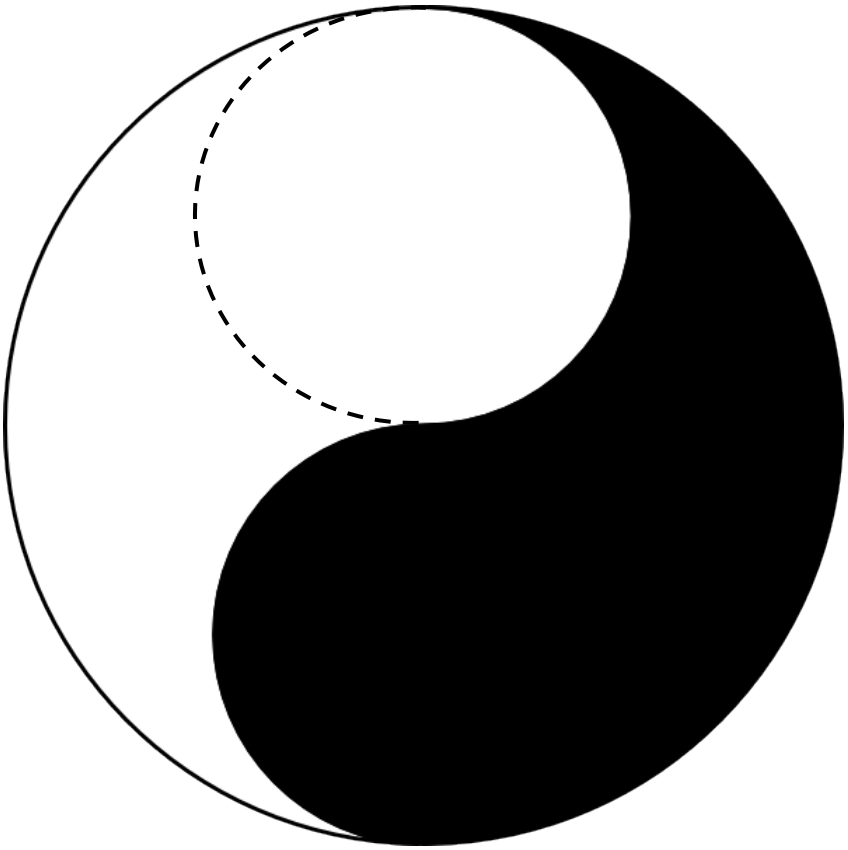
QUESTION SEVEN

YIN AND YANG

THE PROBLEM 6 MARKS

Two mathematicians were dining together at a Chinese restaurant when one noticed the Yin and Yang symbol on the restaurant's menu. He noted how attractive and proportional the symbol was, and then had a very interesting thought… **Is there a way to bisect both the entire symbol and also each of Yang and Yin (black and white respectively)?**

One curved line has been provided to you on the symbol below. Another line of exactly the same shape and length is also required, together with the line of bisection. **Draw these two lines and then use the diagram to prove that both Yin and Yang have been bisected.**



ANSWER SPACE

(please also complete the diagram above by adding the two required lines)

QUESTION SEVEN ANSWER SPACE CONTINUED

QUESTION EIGHT

A SPIRITUAL JOURNEY

THE PROBLEM

4 MARKS

A Buddhist monk leaves to climb a mountain at sunrise one day. He follows a narrow, dirt track which spirals upwards towards the glittering temple at the summit. As he ascends, he stops many times and varies his speed greatly. He arrives at sunset.

After reaching the peak, he stays for many days of fasting and meditation. Then, one day at sunrise, he begins his descent. His speed is again variable and many stops occur. He reaches his original starting point at the base of the mountain by sunset.

Assume the times of sunrise and sunset do not change.

Will there be a point along the monk's route where he is located at exactly the same time on both the ascent and descent?

The answer to this may seem very simply, but the true answer is perhaps not what you expect at all. For one mark, state whether the answer to the question above is yes or no, and for three marks, prove your assertion!

ANSWER SPACE

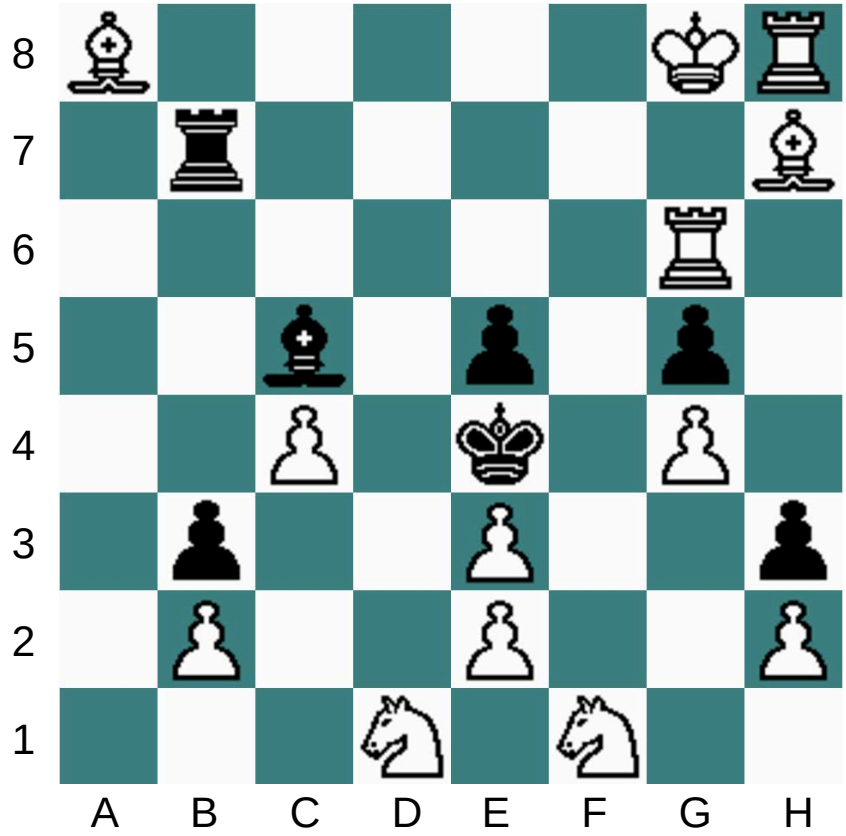
QUESTION EIGHT ANSWER SPACE CONTINUED

QUESTION NINE

UNCHECKMATE

THE PROBLEM 8 MARKS

This famous chess puzzle was devised by German problemist Karl Fabel. Your task is to find a move for **white** that will **NOT** result in checkmate of the black king. You will receive two marks for **identifying** the move and three marks for your **explanation**.



Many past viewers of this problem have complained that there is one element of the layout of pieces above which is **not possible.** While they are incorrect, it is understandable that such a complaint would be raised. For an **extra four marks,** can you **identify** the complaint and **explain** why it is incorrect?

ANSWER SPACE