KNOX GRAMMAR SCHOOL

STATE

DA VINCI DECATHLON 2018

CELEBRATING THE ACADEMIC GIFTS OF STUDENTS IN YEARS 9, 10 & 11

MATHEMATICS

TEAM NUMBER _____

1	2	3	4	5	6	7	8	9	Total	Rank
/4	/4	/7	/4	/6	/6	/5	/5	/9	/50	

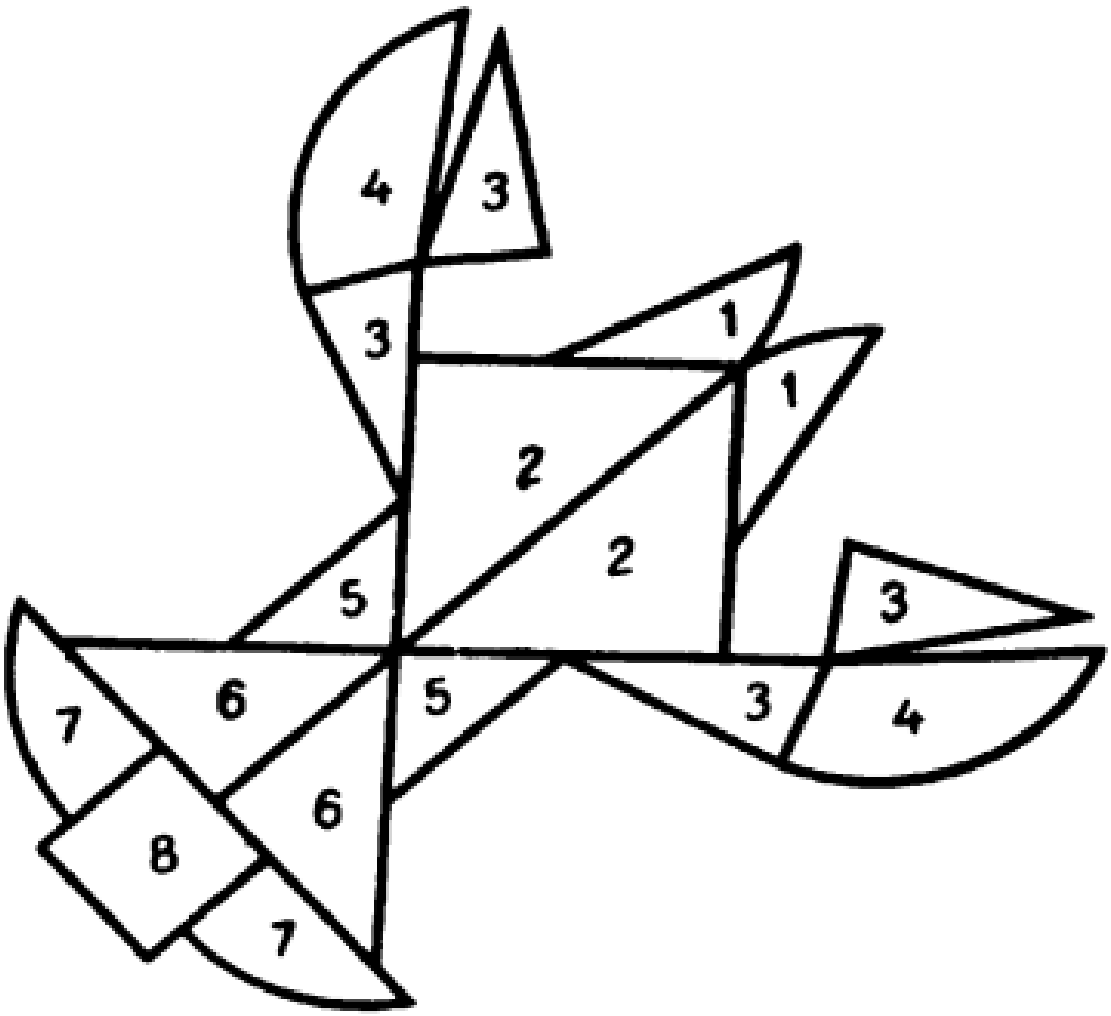
QUESTION ONE

ABSOLUTELY CRAY-ZY

THE PROBLEM 4 MARKS

Below is a **crayfish** made out of 17 numbered shapes. Your task is to rearrange these shapes, in the same proportions, to form a **square** and a **circle.** Make sure to write the numbers on each shape in your answers.

You do not have to cut out the shapes. Just redraw them in the answer space on the following page, making sure to measure the length in order to ensure proportionality. **Pencil** is highly recommended!



QUESTION ONE ANSWER SPACE

QUESTION TWO

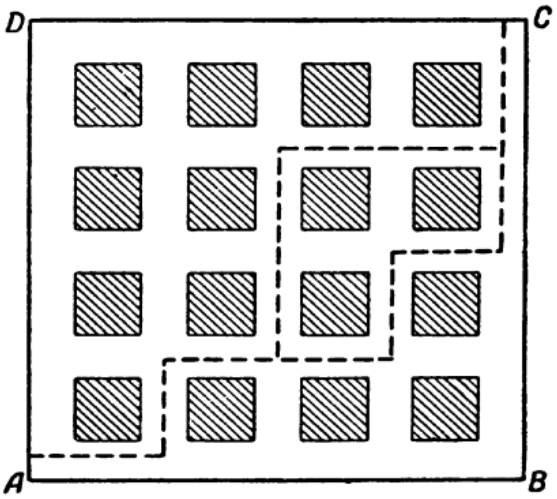
EN ROUTE

THE PROBLEM 4 MARKS

In a freshly designed city, there are 16 identical blocks. The area forms a perfect square. The **postman** for this new city needs to learn **every single possible route** from the post office, marked as A, to the city's exit, marked as C, so that no delivery trip is **unexpected.**

How many possible routes are there? Explain how you came to this answer.

You are only to move upwards and right. Consider using a **diagram** for your explanation.



ANSWER SPACE

QUESTION TWO ANSWER SPACE CONTINUED

QUESTION THREE

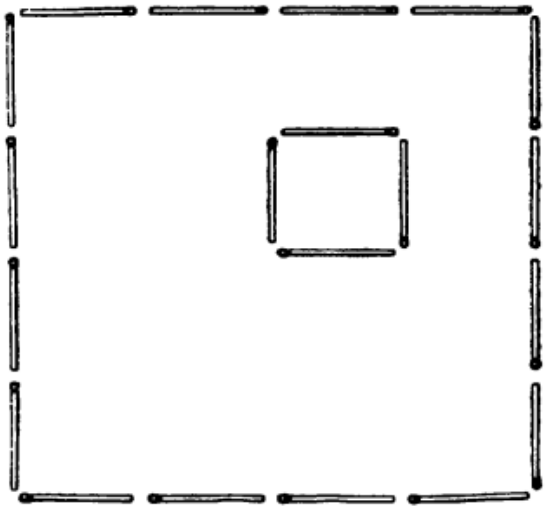
TOUGH MATCH

THE PROBLEM 7 MARKS

Match problems are among the most loved in the worlds of mathematics and puzzles, but also the most challenging. Can you solve these ones? At least one of the answers is not what you might **expect**…

Part One (1 mark)

Using ten more matches, divide the space surrounding the square into five areas of identical size and shape. You can draw on the diagram provided.



Part Two (2 marks)

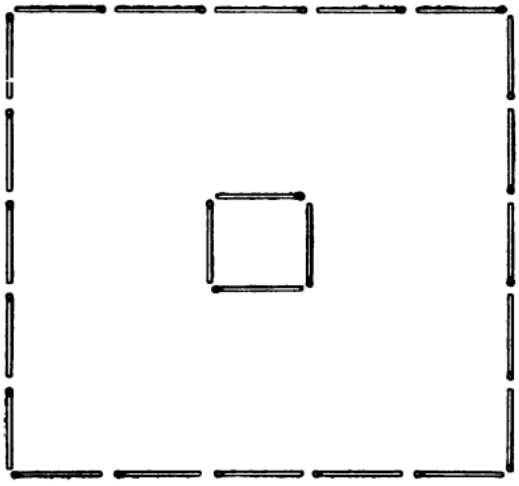
Now, do the same, but using:

(a) 18 matches to form 6 identical areas; and

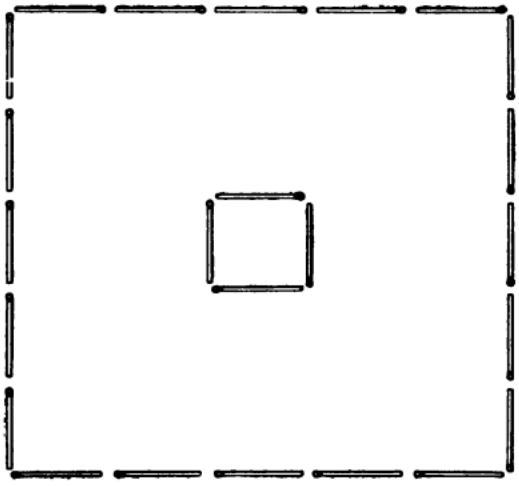
(b) 20 matches to form 8 identical areas.

Again, please use the two diagrams below for your answer.

a)



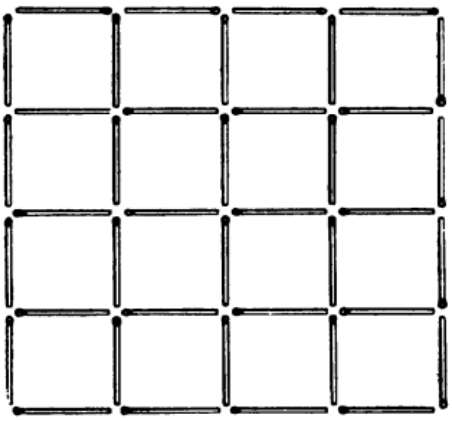
b)



Part Three (1 mark)

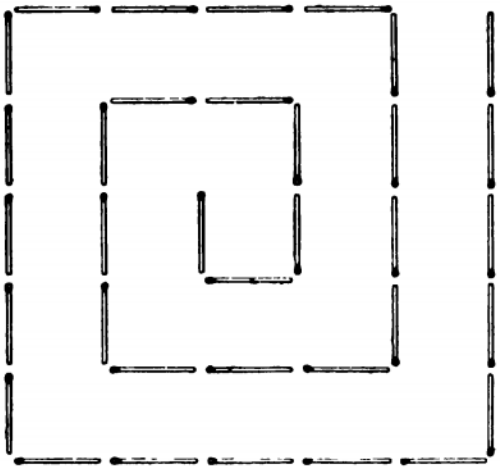
What is the fewest number of matches to remove so that no squares of any size are left in the figure below?

Answer: _____



Part Four (1 mark)

Move four matches of this spiral to form an arrangement of three squares. You will need to redraw your answer to the right of the original figure.



Part Five (2 marks)

Thirteen matches of two inches in length each can be put together to form one ‘yard’. How?

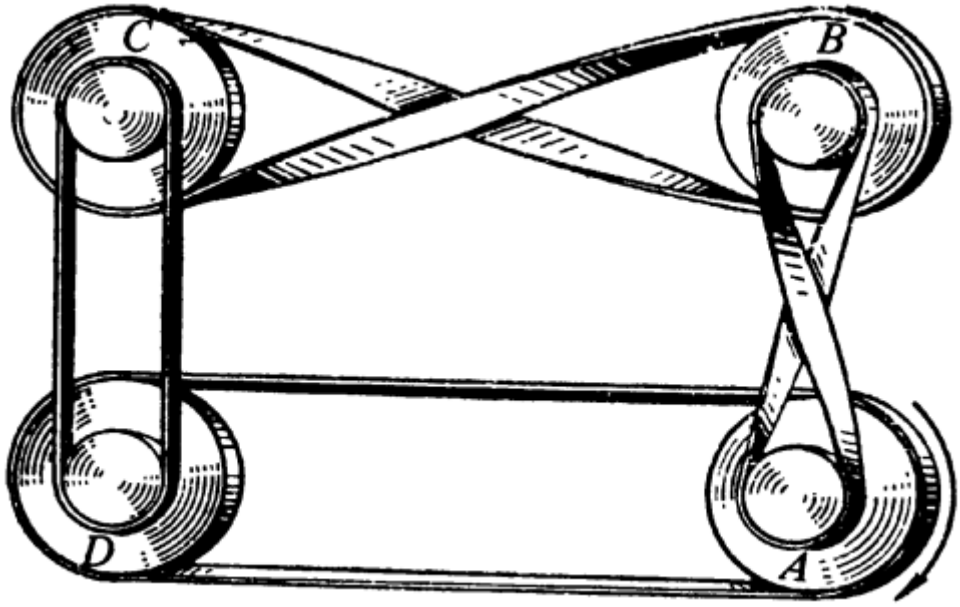
Draw your answer below.

QUESTION FOUR

DIZZYING

THE PROBLEM 4 MARKS

Four belts marked A, B, C and D and connected as shown in the **diagram** below. A begins to rotate **clockwise** as indicated by the arrow. Use this information to answer the series of questions provided.



QUESTIONS

1. Will all four belts rotate based on the diagram above?

   _____

2. Which way does each belt rotate (clockwise, anticlockwise or no rotation)?

   B - _____

   C - _____

   D - _____

3. If all four belts are crossed, like the one between B and C, will all still rotate?

   _____

4. If the following numbers of belts are crossed, will all still rotate?

   1 - _____

   3 - _____

QUESTION FIVE

THINK INSIDE OF THE BOX

THE PROBLEM

6 MARKS

One cubical box houses **27** identical footballs. Another box of the same size contains **64** smaller footballs. The **boxes** are made of the same material, and both are filled to the top.

(not an accurate diagram)

The layers of balls in each box are equal in number, and the outside layers touch the sides of each box. Fundamentally, however, there are **many more footballs** in one box, albeit they are a size smaller.

An employee working at Guerrilla Sports is tasked with determining the **weight** of the two boxes. The results surprise him!

(1) Which is **heavier?** Show full, detailed proof.

(2) Is there a general rule for **cubic numbers** of footballs in boxes? If so, state it.

ANSWER SPACE

QUESTION FIVE ANSWER SPACE CONTINUED

QUESTION SIX

LOST FOR WORDS

THE PROBLEM

6 MARKS

This task is simple. Solve the following equations for the words below.

$$a^2 = bd \text{ and } ad = b^2c$$

B E

W O W

W H A T

W E I R D

S U D D E N

F E A R I N G

S U R P R I S E

U N P L A N N E D

U N E X P E C T E D

U N P R E D I C T E D

A D V E N T I T I O U S

U N A N T I C I P A T E D

R I D I C U L O U S N E S S

E X T R A O R D I N A R I L Y

ANSWER SPACE

QUESTION SIX ANSWER SPACE CONTINUED

QUESTION SEVEN

THE GREAT DIVIDE

THE PROBLEM 5 MARKS

Another short and sweet one. Prove than all **nine-digit** numbers whose three triplets of digits have a sum of the form **AAA** are divisible by **37.**

ANSWER SPACE

QUESTION EIGHT

THE MAGIC NUMBER

THE PROBLEM 5 MARKS

You will be given a handful of **hints** below. Use them to answer the following **question**:

What is so unexpectedly special about the number 6174?

There are two parts to the answer.

HINTS

1. 

2. 0481, not 481.
3. Eventually.
4. a, b, c, d

   a, a, b, c

   a, a, b, b

   a, a, a, b

ANSWER SPACE

QUESTION NINE

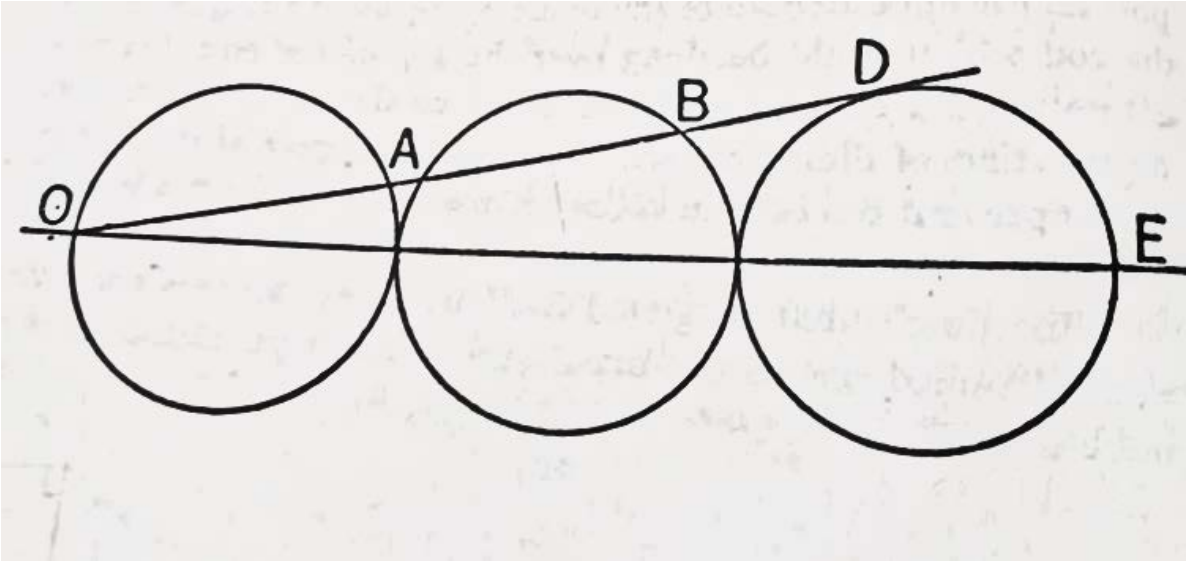
GOING AROUND IN CIRCLES

THE PROBLEM 9 MARKS

Three **circles** of equal radius, *r,* are drawn tangential to each other. Their centres all fall on the line OE. OD is a tangent of the right-most circle, extending to that circle from the outer intersection of OE and the left-most circle. AB is the chord in the middle circle formed by OD.

Determine the **length of AB** in terms of *r.*

Some **construction** will be required.



ANSWER SPACE

QUESTION NINE ANSWER SPACE CONTINUED